Printed Page:- 04 **Subject Code:- ACSBS0205**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2023 - 2024)

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

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SECTION-A 20

1. Attempt all parts:-

1-a. *For what value of* x, *matrix* $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ *is a singular matrix?* (CO1) 1

(a) 1
(b) 2
(c) -2
(d) none of these

1-b. If A is a square matrix such that $A^2 + I = 0$, then A equals (CO1) 1

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$
(d) none of these

1-c. If the rank of a matrix A is 2, then the rank of A' is (CO2) 1

(a) 3
(b) 2
(c) 8

(d) none of these

1-d. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ is (CO2) 1

(a) 2
(b) 3
(c) 4
(d) None of these

1-e. Which of the set of vectors are linearly dependent? (CO3) 1

(a) (1, 1, 4), (1, 0, 0), (1, 1, 0)
(b) (1, 2, 4), (1, 0, 0), (0, 1, 0), (0,0, 1)
(c) (1, 2, 4), (1, -1, 0), (0, 0, 1)
(d) None of these

1-f. The null space of linear transformation from R^3 into R^3 defined as (CO3) 1

(a) (1, 2, 3)
(b) (1, 0, 0)
(c) (0, 1, 0)
(d) (0, 0, 0)

1-g. A square matrix A is positive semi definite if it is symmetric and CO 4 1

(a) $x^T Ax \geq 0$
(b) $x^T Ax \leq 0$
(c) $x^T Ax = 0$
(d) None of these

1-h. If A is skew-Hermitian matrix, then iA is (CO4) 1

(a) Skew-Hermitian matrix
(b) Hermitian matrix
(c) Symmetric matrix
(d) None of these

1-i. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ then Eigen values of A^3 .(CO5) 1

(a) 1,4
(b) 1,64
(c) 4,4
(d) None of these

1-j. If 0 is a Eigen value of a matrix iff the matrix is (CO5) 1

(a) Non singular

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(b) Unitary

(c) Singular

(d) None of these

2. Attempt all parts:-

2.a. Express the matrix $\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (CO1) 2

2.b. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (CO2) 2

2.c. Show that the vectors $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ in R^3 is linearly independent. (CO3) 2

2.d. For the matrix A, Find the sum of eigen values where $A = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. (CO4) 2

2.e. In singular value decomposition if $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ then find S ? (CO5) 2

SECTION-B 30

3. Answer any <u>five</u> of the following:-

3-a. Solve by Cramer's rule: x+y+z =2, 2x+y+3z=9 and x-3y+z=10 (CO1) 6

3-b. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ verify that $(AB)C = A(BC)$ and $A(B+C) = AB + BC$. (CO1) 6

3-c. Show that the vectors $X_1 = [2\ 3\ 1\ -1]$, $X_2 = [2\ 3\ 1\ -2]$, $X_3 = [4\ 6\ 2\ 1]$ are linearly dependent. (CO 6

Express one of the vectors as a linear combination of others.

3-d. **Find the values of a and b such that the rank of matrix** $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ **is 2.**(CO2) 6

3.e. If u and v are any two vectors in an inner product space V. show that $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$. 6

3.f. **Find the eigenvalues and eigenvectors of a matrix of** $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$. (CO4) 6

3.g. **Find a singular value decomposition of the matrix** $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.(CO5) 6

SECTION-C 50

4. Answer any <u>one</u> of the following:-

4-a. Solve the system of equations by matrix method: (CO1) 10
x+2y-3z=4, 2x+3y+2z=2 and 3x-3y-4z=11.

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4-b. If $A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ and adj(adj. A) = A, find a. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Determine the value of λ and μ so that the equations x+y+z=6, x+2y+3z=10, x+2y+λz=μ have
(i) no solution, (ii) a unique solution and (iii) infinite many solutions. (CO2) 10

5-b. 5-b For what values of 'k', the equations x+y+z=1, x+y+4z=k, 4x+y+10z= k^2 have a solution and solve them completely in each case. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Show that the transformation T: $V_2(R) \to V_3(R)$ defined as T(a, b)=(a+b, a-b, b) ∀ a,b ∈ R is linear. Find its null space, nullity, range and rank.(CO3) 10

6-b. Apply Gram- Schmidt process to transform the basis {(1,1,1), (0,1,1), (0,0,1)} into an orthonormal basis.(CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Find the eigen values and eigen vector of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -2 & -8 \\ 0 & -5 & 1 \end{bmatrix}$. (CO4) 10

7-b. Show that the matrix $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is an unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$.
CO 4 10

8. Answer any <u>one</u> of the following:-

8-a. Find the singular values of the $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the SVD decom 10

8-b. Given the following data, use PCA to reduce the dimension from 2 to 1.(CO5) 10

Feature	Example 1	Example 2	Example 3	Example 4
x:	4	8	13	7
y:	11	4	5	14

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